



EE 232 Lightwave Devices Lecture 16: Polarization Dependence

Reading: Chuang, Sec. 4.1-4.2, 9.5
(There is also a good discussion in Coldren,
Appendix 8 and 10)

Instructor: Ming C. Wu

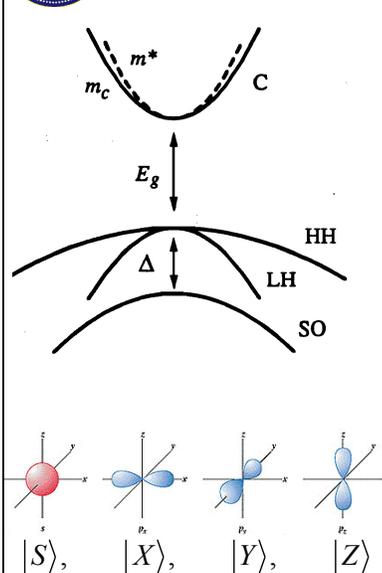
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Electrical Engineering and Computer Sciences Dept.

EE232 Lecture 1

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Detailed Band Structure



Block wavefunction: $\psi_{nk}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} u_{nk}(\vec{r})$

At band edges, $\vec{k} = 0$

Conduction Band: Remnant of s atomic orbital

$$u_{c0}(\vec{r}) = |S\rangle$$

Valence Band: remnants of the p orbitals:

$$u_{v0}(\vec{r}) = |X\rangle, |Y\rangle, |Z\rangle$$

$|S\rangle$ is symmetric in x , y , and z

$|X\rangle$ is anti-symmetric in x , and symm in y , z

$$\langle S|X\rangle = \langle S|Y\rangle = \langle S|Z\rangle = 0$$

$$\langle S|P_x|X\rangle = \langle S|P_y|Y\rangle = \langle S|P_z|Z\rangle \neq 0$$

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k·P Method

$$H\psi_{nk}(\vec{r}) = \left[-\frac{\hbar^2}{2m_0}\nabla^2 + V\right]\psi_{nk}(\vec{r}) = E_n(\vec{k})\psi_{nk}(\vec{r})$$

$$\psi_{nk}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}}u_{nk}(\vec{r})$$

$$u_{nk}(\vec{r} + \vec{R}) = u_{nk}(\vec{r}) \quad \text{periodic function, } \vec{R} \text{ is lattice vector}$$

$$\left[\frac{P^2}{2m_0} + \frac{\hbar}{m_0}\vec{k}\cdot\vec{P} + V(\vec{r})\right]u_{nk}(\vec{r}) = \left[E_n(\vec{k}) - \frac{\hbar^2k^2}{2m_0}\right]u_{nk}(\vec{r})$$

Near $\vec{k} = 0$, $\vec{k}\cdot\vec{P}$ can be treated as a perturbation

$$\left[H_0 + \frac{\hbar}{m_0}\vec{k}\cdot\vec{P}\right]u_{nk}(\vec{r}) = \left[E_n(\vec{k}) - \frac{\hbar^2k^2}{2m_0}\right]u_{nk}(\vec{r})$$

$$\text{Kane's P parameter: } P = \frac{-i\hbar}{m_0}\langle S|P_z|Z\rangle$$

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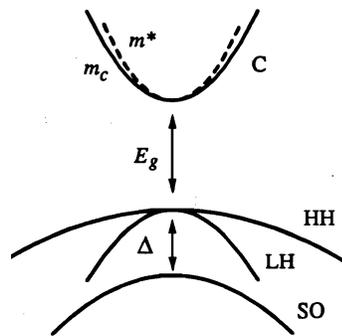
Eigenvectors

Define z-direction along \vec{k} (i.e., $\vec{k} = k\hat{z}$)

CB wavefunctions:

$$u_c = |iS \uparrow\rangle$$

$$\bar{u}_c = |iS \downarrow\rangle$$



VB wavefunctions:

$$u_{hh} = \left|\frac{3}{2}, \frac{3}{2}\right\rangle = \frac{-1}{\sqrt{2}}|(X+iY)\uparrow\rangle$$

$$\bar{u}_{hh} = \left|\frac{3}{2}, \frac{-3}{2}\right\rangle = \frac{1}{\sqrt{2}}|(X-iY)\downarrow\rangle$$

$$u_{lh} = \left|\frac{3}{2}, \frac{1}{2}\right\rangle = \frac{-1}{\sqrt{6}}|(X+iY)\downarrow - 2Z\uparrow\rangle$$

$$\bar{u}_{lh} = \left|\frac{3}{2}, \frac{-1}{2}\right\rangle = \frac{1}{\sqrt{6}}|(X-iY)\uparrow + 2Z\downarrow\rangle$$

$$u_{so} = \left|\frac{1}{2}, \frac{1}{2}\right\rangle = \frac{1}{\sqrt{3}}|(X+iY)\downarrow + Z\uparrow\rangle$$

$$\bar{u}_{so} = \left|\frac{1}{2}, \frac{-1}{2}\right\rangle = \frac{1}{\sqrt{3}}|(X-iY)\uparrow - Z\downarrow\rangle$$

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Second-Order Perturbation

Conduction Band:

Second-order perturbation:

$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + \frac{\hbar^2}{m_0^2} \sum_{n \neq c} \frac{|\vec{k} \cdot \vec{P}_{cn}|^2}{E_c(0) - E_n(0)}$$

$$\vec{P}_{cn} = \langle u_c | \vec{P} | u_n \rangle, \quad n = hh, lh, \text{ so}$$

Use Kane's P parameter:

$$P = \frac{-i\hbar}{m_0} \langle S | P_z | Z \rangle$$

$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + k^2 P^2 \left(\frac{2}{3} \frac{1}{E_g} + \frac{1}{3} \frac{1}{E_g + \Delta} \right)$$

$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + \frac{k^2 P^2}{3} \left(\frac{3E_g + 2\Delta}{E_g(E_g + \Delta)} \right) = \frac{\hbar^2 k^2}{2m_e^*}$$

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Eigenvalues

Conduction Band:

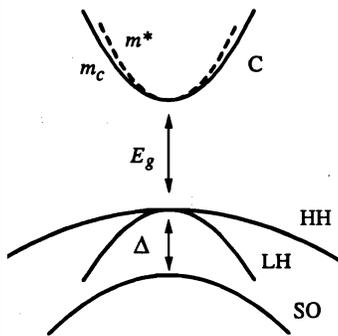
$$E_c(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} + \frac{k^2 P^2}{3} \left(\frac{3E_g + 2\Delta}{E_g(E_g + \Delta)} \right) = \frac{\hbar^2 k^2}{2m_e^*}$$

Valence Band:

$$\text{HH: } E_{hh}(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} \quad (\text{incorrect in this approx})$$

$$\text{LH: } E_{lh}(\vec{k}) = \frac{\hbar^2 k^2}{2m_0} - \frac{2k^2 P^2}{3E_g} = -\frac{\hbar^2 k^2}{2m_{lh}^*}$$

$$\text{SO: } E_{so}(\vec{k}) = -\Delta + \frac{\hbar^2 k^2}{2m_0} - \frac{2k^2 P^2}{3(E_g + \Delta)} = -\Delta + -\frac{\hbar^2 k^2}{2m_{so}^*}$$



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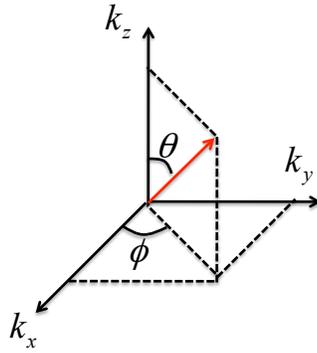
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Wavefunctions in General Coordinates

When \vec{k} is not along z direction (i.e., $\vec{k} \neq k\hat{z}$)

$$\vec{k} = k \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} + k \cos \theta \hat{z}$$



The new wavefunctions are now linear combinations of new orbital functions $|X'\rangle, |Y'\rangle, |Z'\rangle$. They can be transformed back to the orbital functions in the fixed coordinate through the following

Coordination Transformation:

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \theta & \cos \phi & 0 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

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Example: Heavy Hole Wavefunction

$$\begin{bmatrix} X' \\ Y' \\ Z' \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ -\sin \theta & \cos \phi & 0 \\ \sin \theta \cos \phi & \sin \theta \sin \phi & \cos \theta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

$$\begin{aligned} u'_{hh} &= \left| \frac{3}{2}, \frac{3}{2} \right\rangle = \frac{-1}{\sqrt{2}} |(X' + iY') \uparrow'\rangle = \frac{-1}{\sqrt{2}} |(X' + iY') \uparrow'\rangle \\ &= \frac{-1}{\sqrt{2}} |(\cos \theta \cos \phi - i \sin \theta)X + (\cos \theta \sin \phi + i \cos \theta)Y - (\sin \theta)Z \rangle \uparrow'\rangle \end{aligned}$$

$$\begin{aligned} u'_{hh} &= \left| \frac{3}{2}, \frac{-3}{2} \right\rangle = \frac{-1}{\sqrt{2}} |(X' - iY') \uparrow'\rangle = \frac{-1}{\sqrt{2}} |(X' - iY') \uparrow'\rangle \\ &= \frac{1}{\sqrt{2}} |(\cos \theta \cos \phi + i \sin \theta)X + (\cos \theta \sin \phi - i \cos \theta)Y - (\sin \theta)Z \rangle \uparrow'\rangle \end{aligned}$$

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C-HH Matrix Element

Optical Matrix Element:

$$H'_{ba} = -\frac{eA_0}{2m_0} \hat{e} \cdot \langle b | \vec{P} | a \rangle = -\frac{eA_0}{2m_0} \hat{e} \cdot \vec{p}_{cv} \cdot I_{hm}^{en}$$

There are 4 possible terms for C-HH transition: $\overline{M}_{C-HH} = \overline{p}_{C-HH}$

$$\langle \hat{u}'_c | \vec{P} | \hat{u}'_{hh} \rangle = \frac{-1}{\sqrt{2}} \left[(\cos\theta \cos\phi - i \sin\phi) P_x \hat{x} + (\cos\theta \sin\phi + i \cos\phi) P_y \hat{y} - (\sin\theta) P_z \hat{z} \right]$$

$$= \frac{-P_x}{\sqrt{2}} \left[(\cos\theta \cos\phi - i \sin\phi) \hat{x} + (\cos\theta \sin\phi + i \cos\phi) \hat{y} - (\sin\theta) \hat{z} \right]$$

$$\langle \hat{u}'_c | \vec{P} | \hat{u}'_{hh} \rangle = 0 \quad \because \langle \uparrow' | \downarrow' \rangle = 0$$

$$\langle \hat{u}'_c | \vec{P} | \hat{u}'_{hh} \rangle = 0$$

$$\langle \hat{u}'_c | \vec{P} | \hat{u}'_{hh} \rangle = \frac{P_x}{\sqrt{2}} \left[(\cos\theta \cos\phi + i \sin\phi) \hat{x} + (\cos\theta \sin\phi - i \cos\phi) \hat{y} - (\sin\theta) \hat{z} \right]$$

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C-HH Matrix Element

To find polarization Dependence \Rightarrow Integrate over all possible θ and ϕ :

$$\left| \hat{x} \cdot \overline{p}_{C-HH} \right|^2 = \frac{1}{4\pi} \int_0^\pi \sin\theta \cdot d\theta \int_0^{2\pi} d\phi \cdot$$

$$\frac{1}{2} \left\{ \frac{P_x^2}{2} (\cos^2\theta \cos^2\phi + \sin^2\phi) + \frac{P_x^2}{2} (\cos^2\theta \cos^2\phi + \sin^2\phi) \right\}$$

$$\left| \hat{x} \cdot \overline{p}_{C-HH} \right|^2 = \frac{1}{4\pi} \int_0^\pi \sin\theta \cdot d\theta \int_0^{2\pi} d\phi \cdot \left\{ \frac{P_x^2}{2} (\cos^2\theta \cos^2\phi + \sin^2\phi) \right\} = \frac{P_x^2}{3} \equiv M_b^2$$

Similarly,

$$\left| \hat{y} \cdot \overline{p}_{C-HH} \right|^2 = M_b^2$$

$$\left| \hat{z} \cdot \overline{p}_{C-HH} \right|^2 = M_b^2$$

C-HH transition in Bulk Semiconductor is Polarization Independent!

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C-LH Matrix Element

Similarly:

$$\left| \hat{x} \cdot \vec{p}_{C-LH} \right|^2 = M_b^2$$

$$\left| \hat{y} \cdot \vec{p}_{C-LH} \right|^2 = M_b^2$$

$$\left| \hat{z} \cdot \vec{p}_{C-LH} \right|^2 = M_b^2$$

C-LH transition in Bulk Semiconductor is Polarization Independent!

⇒

Bulk Semiconductor is Polarization Independent!

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Values of Matrix Element

To find polarization Dependence ⇒ Integrate over all possible θ and ϕ :

$$M_b^2 = \frac{P_x^2}{3} = \frac{1}{3} \frac{m_0^2}{\hbar^2} P^2 = \frac{1}{3} \frac{m_0^2}{\hbar^2} \left(1 - \frac{m_e^*}{m_0} \right) \left(\frac{\hbar^2 E_g (E_g + \Delta)}{2m_e^* (E_g + \frac{2}{3}\Delta)} \right)$$

$$\left(1 - \frac{m_e^*}{m_0} \right) \approx 1$$

$$M_b^2 \approx \frac{m_0^2 E_g (E_g + \Delta)}{6m_e^* (E_g + \frac{2}{3}\Delta)} \propto \frac{E_g}{m_e^*}$$

$$M_b^2 \approx \frac{m_0}{6} \left(\frac{m_0 E_g (E_g + \Delta)}{m_e^* (E_g + \frac{2}{3}\Delta)} \right) \rightarrow \frac{m_0}{6} E_p$$

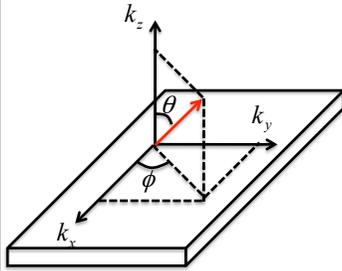
	E_p
GaAs	25.7 eV
InP	20.7 eV
InAs	22.2 eV

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C-HH Matrix Element in Quantum Well



Quantum well is polarization dependent !

Integrate over ϕ only

$$\begin{aligned} \left| \hat{x} \cdot \vec{p}_{C-HH} \right|^2 &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \cdot \left\{ \frac{P_x^2}{2} (\cos^2 \theta \cos^2 \phi + \sin^2 \phi) \right\} \\ &= \frac{3}{4} (1 + \cos^2 \theta) M_b^2 \end{aligned}$$

$$\begin{aligned} \left| \hat{y} \cdot \vec{p}_{C-HH} \right|^2 &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \cdot \left\{ \frac{P_x^2}{2} (\cos^2 \theta \sin^2 \phi + \cos^2 \phi) \right\} \\ &= \frac{3}{4} (1 + \cos^2 \theta) M_b^2 \end{aligned}$$

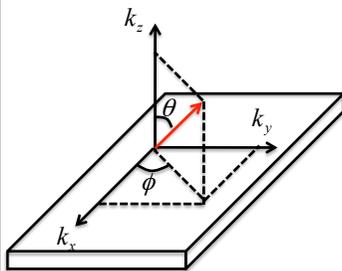
$$\begin{aligned} \left| \hat{z} \cdot \vec{p}_{C-HH} \right|^2 &= \frac{1}{2\pi} \int_0^{2\pi} d\phi \cdot \left\{ \frac{P_x^2}{2} (\sin^2 \theta) \right\} \\ &= \frac{3}{2} (\sin^2 \theta) M_b^2 \end{aligned}$$

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C-LH Matrix Element in Quantum Well



Quantum well is polarization dependent !

Integrate over ϕ only :

$$\left| \hat{x} \cdot \vec{p}_{C-LH} \right|^2 = \left(\frac{5}{4} - \frac{3}{4} \cos^2 \theta \right) M_b^2$$

$$\left| \hat{y} \cdot \vec{p}_{C-LH} \right|^2 = \left(\frac{5}{4} - \frac{3}{4} \cos^2 \theta \right) M_b^2$$

$$\left| \hat{z} \cdot \vec{p}_{C-LH} \right|^2 = \left(\frac{1}{2} + \frac{3}{2} \cos^2 \theta \right) M_b^2$$

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Quantum Well Matrix Element

	TE $\hat{e} = \hat{x}$ or \hat{y}	TM $\hat{e} = \hat{z}$	All Polarizations (2xTE + TM)
C-HH Transition	$\frac{3}{4}(1 + \cos^2 \theta)M_b^2$	$\frac{3}{2}(\sin^2 \theta)M_b^2$	$3M_b^2$
C-LH Transition	$\left(\frac{5}{4} - \frac{3}{4}\cos^2 \theta\right)M_b^2$	$\left(\frac{1}{2} + \frac{3}{2}\cos^2 \theta\right)M_b^2$	$3M_b^2$
Sum Rule: HH + LH	$2M_b^2$	$2M_b^2$	$6M_b^2$

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Angular Factor

Angular factor: $\cos^2 \theta = \frac{k_z^2}{k^2} = \frac{k_z^2}{k_t^2 + k_z^2}$

In quantum wells, k_z is quantized: $E_{en} = \frac{\hbar^2 k_z^2}{2m_e^*}$

$$\cos^2 \theta = \frac{\frac{\hbar^2 k_z^2}{2m_e^*}}{\frac{\hbar^2 k_t^2}{2m_e^*} + \frac{\hbar^2 k_z^2}{2m_e^*}} = \frac{E_{en}}{\frac{\hbar^2 k_t^2}{2m_e^*} + E_{en}}$$

k_t can be estimated from $\frac{\hbar^2 k_t^2}{2m_r^*} = \hbar\omega - E_g - E_{en} - |E_{hm}|$

At band edge, $k_t = 0 \Rightarrow \cos^2 \theta = 1$

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Transition Strength versus Transverse Wavevector

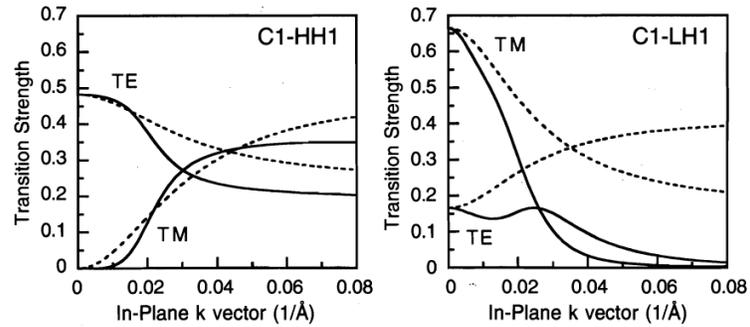


FIGURE A10.5 Relative transition strengths for both TE and TM light polarization for the two lowest subband transitions in an unstrained GaAs/Al_{0.2}Ga_{0.8}As 80Å QW. The dashed curves represent what one would calculate assuming parabolic subbands. The transition strength as plotted here is defined as $|M_T|^2/|M|^2$ (bulk value is 1/3).